

Chapter 7B - Day 1

Guidelines

1. READ the Problem.
2. Define variables, draw a picture.
3. What are you trying to optimize?
Is it a max or min?
4. Write a formula for what you are optimizing. Simplify it to one variable, if necessary.
5. Find the interval over which the one variable can vary.
6. Take the derivative and find critical points
7. Use techniques from Ch 6 to find the min/max.

Ex: What is the largest possible product you can form from 2 non-negative numbers whose sum is 20?

Let x, y be our numbers, $x \geq 0, y \geq 0$

$$x + y = 20$$

$$y = 20 - x \quad \text{thus } \begin{array}{l} y \geq 0 \\ 20 - x \geq 0 \end{array}$$

$$20 \geq x \quad x \in [0, 20]$$

We want to maximize the product $x \cdot y$

$$x \cdot y = x(20 - x)$$

$$\text{let } f(x) = x(20 - x) = 20x - x^2$$

$$f'(x) = 20 - 2x = 2(10 - x)$$

$$f'(x) = 0 \quad \text{when } x = 10$$



We have a continuous function over a closed and bounded interval... use EVT!

$$f(0) = 0(20-0) = 0$$

$$f(10) = 10(20-10) = 100 \leftarrow \text{max}$$

$$f(20) = 20(20-20) = 0$$

largest possible product is $\boxed{100}$

and it occurs when $x=10$ and

$$y = 20 - 10 = 10$$

Ex: Suppose the product of x and y is 36 and both x and y are positive. What is the minimum possible sum of x and y ?

x and y are our numbers, $x > 0$, $y > 0$

$$x \cdot y = 36$$

$$y = \frac{36}{x} > 0 \text{ for all } x > 0$$

$$\text{thus } x \in (0, \infty)$$

We want to minimize $x+y$

$$x+y = x + \frac{36}{x}$$

$$\text{let } f(x) = x + \frac{36}{x} = x + 36x^{-1}$$

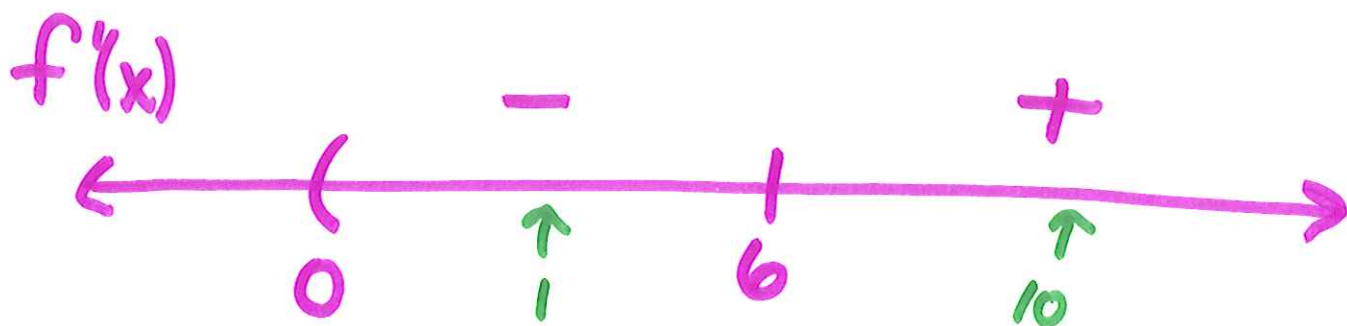
$$f'(x) = 1 - 36x^{-2} = 1 - \frac{36}{x^2}$$

$$1 - \frac{36}{x^2} = 0$$

$$1 = \frac{36}{x^2} \rightarrow x^2 = 36 \rightarrow x = \pm 6$$

-6 not
in
interval

Because we have an open interval, we need to use the 1st derivative test.



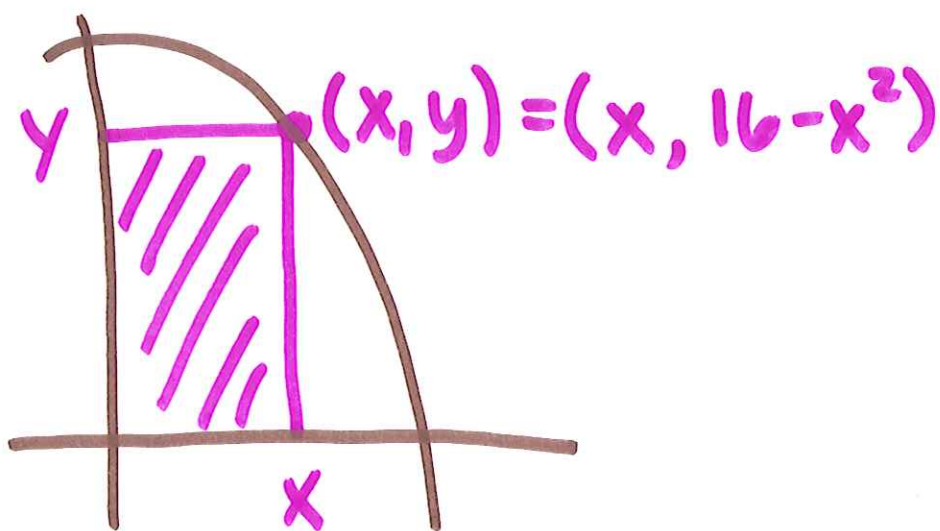
$$f'(1) = 1 - \frac{36}{1^2} = -35 \text{ " - "}$$

$$f'(10) = 1 - \frac{36}{10^2} = \frac{64}{100} \text{ " + "}$$

min occurs at $x=6$

$$\begin{aligned} \text{min sum is } x+y &= x + \frac{36}{x} \\ &= 6 + \frac{36}{6} = \boxed{12} \end{aligned}$$

Ex: find the area of the largest rectangle with one corner at the origin, the opposite corner in the 1st quadrant on the graph of the parabola $f(x) = 16 - x^2$ and sides parallel to the axes.



find max area

$$A = x \cdot y = x(16 - x^2) \\ = 16x - x^3$$

$$\text{let } f(x) = 16x - x^3$$

$$\text{then } f'(x) = 16 - 3x^2$$

$$f'(x)=0 \text{ when } 16-3x^2=0$$

$$16=3x^2$$

$$\frac{16}{3}=x^2$$

$$\pm\sqrt{\frac{16}{3}}=x$$

$$\pm\frac{4}{\sqrt{3}}=x$$

find interval \rightarrow we're in 1st quadrant!

$$x \geq 0$$

$$y=16-x^2 \geq 0$$

$$16 \geq x^2$$

$$-4 \leq x \leq 4$$

thus $x \in [0, 4]$

$-\frac{4}{\sqrt{3}}$ not in interval

f continuous over a closed and bounded interval - use EVT!

$$f(0) = 0(16 - 0^2) = 0$$

$$f\left(\frac{4}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} \left(16 - \left(\frac{4}{\sqrt{3}}\right)^2\right) = \frac{4}{\sqrt{3}} \left(16 - \frac{16}{3}\right)$$
$$= \frac{4}{\sqrt{3}} \cdot \frac{32}{3} = \frac{128}{3\sqrt{3}}$$

↑ max

$$f(4) = 4(16 - 4^2) = 0$$

max occurs at $x = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

and max area is $\frac{128}{3\sqrt{3}} = \boxed{\frac{128\sqrt{3}}{9}}$